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COMBINATORICS

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Combinatorics is the newsletter of the Combinatorial Mathematics Society of Australasia, and is published by the Society. Annual subscription rate for individual non-members and institutions is $A4. All enquiries should be directed to the C.M.S.A., C/- Dr. Rey Casse, Department of Pure Mathematics, University of Adelaide, Box 498, G.P.O., Adelaide, S.A.5001, Australia.

GENERAL ANNOUNCEMENTS

Combinatorics Conference :

British Combinatorial Conference, 11th - 15th July, 1983, Southampton, Hampshire, England. Further details from Dr. A.D. Keedwell, Department of Mathematics, University of Surrey, Guildford, Surrey, GU2 5XH.

Tenth Australian Conference on Combinatorial Mathematics :

The Tenth Australian Conference on Combinatorial Mathematics will be conducted by the Combinatorial Mathematics Society of Australasia at the University of Adelaide, Australia, from Monday, 23rd August, to Friday, 27th August, 1982. The fourth Annual General Meeting of the C.M.S.A. will be held at the conference. Registration will take place on the Sunday evening and on the Monday morning before the formal programme begins.

Invited Addresses

Not all details are available. Among those who have agreed to give a one-hour invited talk are:

Dr. D. Keedwell, University of Surrey, U.K.
Dr. J. Hirschfeld, University of Sussex, U.K.

We hope also to have invited talks from the following:

Professor J.A. Thas, University of Ghent, Belgium.
Professor A. Barlotti, University of Bologna, Italy.
Professor C.C. Lindner, Auburn University, U.S.A.
Professor N. Pullman, Queen's University, Canada.
Professor R. Stanton, University of Manitoba, Canada.

There will also be some expository lectures.

Contributed Papers

Papers are welcome in all areas of pure and applied combinatorics. Contributed talks will be allowed thirty minutes each, five minutes of which is usually for discussion. If you wish to give a paper, please insert the title on the reply form and send a typed abstract by 30th June, 1982.

Conference Proceedings

We expect Springer-Verlag to publish the refereed proceedings of the conference in the Lecture Notes in Mathematics series. Prospective authors are advised that the series editors of LNIM require that "The contributions should be of a high standard and of current interest. Contributions should contain sufficient information and motivation. Except possibly in the case of major surveys, they should Present at least outlines of the proofs, enough to enable an expert to complete them. Thus resumés and mere announcements of papers appearing elsewhere cannot be included, although more detailed versions of a contribution may well be published in other places later".

All prospective authors are asked to bring their papers with them to the conference in a form suitable for refereeing, either typed or legibly and neatly handwritten. At the latest, papers will be required in this form, ready for refereeing, by 30 September, 1982. It is hoped that typing of all papers ready for photo-offset will be done in Adelaide, in order to achieve uniformity throughout the volume.

Accommodation

Accommodation has been reserved at Aquinas College, Palmer Place, North Adelaide. The cost will be $15 for bed and breakfast; lunch ($2.50) and dinner ($3.50) will be available if required. The college is a pleasant fifteen minutes walk from the University campus, but some transport will be provided in the mornings.

If college accommodation is not to your taste, there are a number of inner city hotels and motels within walking distance. However, this tends to be extremely expensive.

Would anybody requiring accommodation at Aquinas College, who has special needs (e.g. vegetarian, physical disability) please advise us when booking.

Parking is available at the college, but definitely not on campus. Some short term (parking meter) parking is usually available nearby.

During the week of the conference, delegates will be honorary members of both the University of Adelaide Union and the Staff Club, and facilities of both will be available for lunch and dinner. There are a number of good restaurants within easy walking distance of the University.

Travel

If you wish to be met at Adelaide airport, please include your flight information on the reply form. If you are travelling by car and would like a map of the campus *before* arrival, please ask and we will send you one.

Registration

Registration fees are payable at the conference, and will be:

- $25 for CMSA members;
- $30 for non-members (this gives membership of CMSA for the rest of 1982);
- $12.50 for student members or unemployed members of CMSA;
- $15 for non-member students or unemployed people (giving membership of CMSA for the rest of 1982).

Social Programme

There will be a wine and cheese evening with Registration after dinner on the evening of Sunday, 22nd August, in Aquinas College.

Other social activities are being organised and details will be notified in the next newsletter.

The Conference dinner will be on Thursday evening in the North and South dining rooms of the University Union. The cost is expected to be around $15.

Eleventh Australian Conference on Combinatorial Mathematics, to be held at the University of Canterbury, provisional dates 29th August-2nd September, 1983. For further details contact Dr. D.R. Breach, Department of Mathematics, University of Canterbury, Christchurch, New Zealand.

Subscription Renewal

Members are reminded that their 1982 subscriptions fell due on 1 January. The rates remain at $A4 for those in full-time employment and $A2 otherwise. *If a tick appears in the box above,* your subscription had not been received at time of mailing. Prompt payment would be appreciated. Please make cheques, etc., payable to C.M.S.A. and post with full name, address, and telephone number to: Dr. L.R.A. Casse, Director, CMSA, Department of Pure Mathematics, University of Adelaide, Adelaide, S.A. 5001.

This issue of *Combinatorics* is being sent to all 1981 and 1982 members. The membership list will be pruned of unfinancial members before the next issue.

Receipts are enclosed for those paid up members who have not already received them. Because of the costs of cashing cheques in $U.S., would those members who wish to pay in cheques in $U.S. please send $U.S.6.00. Would those who have underpaid please add $U.S.1.00 to next year's subscription.

GENERAL ANNOUNCEMENTS

Titbits for publication can be sent to Rey Casse, Pure Mathematics Department, University of Adelaide, Box 498, G.P.O., Adelaide, 5001.

PROBLEM SECTION

Here is a partial non solution to the problem posed in Vol. 3 (4).

EULERIAN CIRCUITS IN K_n

Brendan D. McKay

Let $\mathrm{Eul}(K_{2n+1})$ be the number of Eulerian circuits in K_{2n+1}, where two such circuits are not counted separately if one is just a cyclic permutation of the other. The problem of determining $\mathrm{Eul}(K_{2n+1})$ was posed by O. Ore [Theory of Graphs, AMS (1962)] but not solved there. A solution was claimed by V. A. Sorokin [A formula for the number of Euler cycles of a complete graph, *Uspehi Mat. Nauk* 24 (1969) 6 (150) 191–192] and, even though it is not correct, the method is interesting anyway. My discussion of it is based mostly on the abstract in Math. Reviews (141 #5246), so if it is up the creek blame the reviewer, a certain Wilfried someone.

(1) If we orient the edges of K_{2n+1} in the direction they are traversed by a particular Eulerian circuit we get a ***regular tournament*** — one in which every vertex has in-degree = out-degree = n.

(2) The number of (labelled) regular tournaments with $2n+1$ vertices is

$$B_{2n+1} = \frac{(2n)!\,(2n-2)!\cdots 2!\,0!}{(n)!\,(n-1)!\cdots 1!\,0!}.$$

(3) The number A_{2n+1} of (directed) Eulerian circuits of a regular tournament can be found using a result of T. van Aardenne-Ehrenfest and N. G. de Bruijn [Circuits and trees in oriented linear graphs, *Simon Stevin* 28 (1951) 203–217]. In fact it is $(n-1)!^{2n+1}$ times the number of spanning arborescences of the tournament with a specified root. The latter can be found using a determinant formula of W. T. Tutte [The dissection of equilateral triangles into equilateral triangles, *Proc. Cambridge Philos. Soc.* 44 (1948) 463–482].

(4) Putting this together, $\mathrm{Eul}(K_{2n+1}) = A_{2n+1}B_{2n+1}$.

Unfortunately, there are two major problems. Firstly, the formula for B_{2n+1} is wrong, as shown by V. A. Liskovec [The number of Eulerian digraphs and homogeneous tournaments, *Vescī Akad. Navuk BSSR Ser. Fīz.-Mat. Navuk* 1 (1971) 22–27]. A more serious error is the assumption that A_{2n+1} is independent of the structure of the tournament — false for $n \geq 7$.

Despite the flaws, the method can be used to compute $\mathrm{Eul}(K_{2n+1})$ for small n. Let $\mathcal{T}$ be the set of all labelled regular tournaments with $2n+1$ vertices. For

2n+1

$T \in \mathcal{T}$, let $\kappa(T)$ be the number of spanning arborescences of T with specified root (independent of the choice of root!). Then $\mathrm{Eul}(K_{2n+1}) = (n-1)!^{n} \sum_{T \in \mathcal{T}} \kappa(T)$.

n	B_{2n+1}	$\mathrm{Eul}(K_{2n+1})$
1	2	2
2	24	264
3	2640	1299 76320
4	32 30080	911 52005 70212 35200
5	4 82515 08480	257 32699 92380 92967 42778 51601 30560

I have not seen the number $\mathrm{Eul}(K_{n,n})$ even mentioned anywhere, but computation for $n \le 8$ should be feasible using much the same method. In both cases the general problem seems to still be open.

Further notes:

(1) A general inequality given by W. T. Tutte [Connectivity in Graphs, Univ. of Toronto Press (1966)] gives

$$2^{n(2n-1)}(n-1)!^{2n+1} \le \mathrm{Eul}(K_{2n+1}) \le \frac{(n-1)!}{2^{n(2n-1)}}\left(\frac{(2n)!}{n!}\right)^{2n},$$

which proves that $\log \mathrm{Eul}(K_{2n+1}) \sim 2n^2 \log n$. The upper bound seems much closer, being within a factor of 1·7 for $1 \le n \le 5$.

(2) The number of directed Eulerian circuits in a complete directed graph $\vec{K}_n$ is $(n-2)!^n n^{n-2}$.

(3) The number of directed Eulerian circuits in a complete directed bipartite graph $\vec{K}_{m,n}$ is $n!^m m!^n/(nm)$.

ABSTRACT OF M.MATHS. THESIS (UNIVERSITY OF NEWCASTLE)

LATTICE POINTS IN PLANAR REGIONS

Raymond Armstrong

The lattice points of a Euclidean space are the points with only integer coordinates. Given any region of a Euclidean space, usually the plane, we are primarily interested in the number of lattice points it contains.

Chapter One reviews some of the established results and proof techniques concerning asymptotic behaviour of the number of lattice points in various convex regions, as well as combinatorial results on the lattice point covering property and related problems.

Chapter Two considers a class of regions, termed simple regions, in which the number of lattice points is essentially equal to the measure, in the sense that the two quantities are asymptotically equal when the region undergoes similarity transformation by a factor that tends to infinity. We prove that certain regions, such as bounded open regions, are simple regions and give examples of non-simple regions.

The covering of lattice points of the plane by closed discs is examined in Chapter Three. We define the critical diameter of a configuration of lattice points: this is the smallest diameter for a closed disc to guarantee that irrespective of the location of the disc, it necessarily covers a set of lattice points in the given configuration. We study a special class of configurations for which the critical diameter can be readily calculated, and then show how to obtain the critical diameter of any given configuration from that of the smallest member of this special class which contains a subset congruent to the given configuration. We also define the critical diameter for n lattice points as the smallest diameter for a closed disc to guarantee that irrespective of the location of the disc, it necessarily covers at least n lattice points, and tabulate the values for $n \leq 20$.

The covering diameter of a configuration is the smallest size of disc capable (with suitably chosen location) of covering the configuration. Again, we study a special class of configurations and show how to determine their covering diameters. From this we show how the covering diameter of any given configuration can be deduced. We also define covering diameter for n lattice points as the smallest size of disc capable (with suitably chosen location) of covering n lattice points. The covering diameter of n lattice points is readily deduced once covering diameters are known for the special class of configurations. We tabulate these values for $n \leq 20$.

The author has also studied similar questions with closed squares replacing closed discs. The Appendix summarises some of the results obtained.

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Professor J. A. Thas, of the University of Ghent, Belgium, has just informed us that he will be in Adelaide for the month of August. He will be accompanied by his wife and two children (5 & 10 yrs.).

During that month, his contact address is

c/- Rey Casse,
Dept. of Pure Maths,
University of Adelaide.
(Phone: 08-2285080)