COMBINATORICS

VOLUME 11 (1989), NUMBER 2, APRIL, PAGES 4–18.

The newsletter of the
Combinatorial Mathematics Society of Australasia

ANNUAL SUBSCRIPTIONS

If there is a tick here, your membership fee is now due.
Please send $5 in Australian currency to the Treasurer of
CMSA (adding $2 to foreign cheques to cover bank charges).

COMBINATORIAL MATHEMATICS SOCIETY

of

AUSTRALASIA

The Combinatorial Mathematics Society of Australasia was formed in 1978, with the aim (as stated in its constitution) of promoting combinatorial mathematics; this has been broadly interpreted as including the relevant areas of computing. It disseminates information about combinatorics and combinatorialists through its newsletter *Combinatorics* and conducts an annual conference with refereed published proceedings. There are currently about 120 members from all over the world.

Any interested person is invited to join the C.M.S.A. Annual subscription for 1989 is Australian $5, payable to C.M.S.A. Members receive the newsletter and a reduction in the conference registration fee. Please address all enquiries, giving your full name and address, to Diane Donovan or Elizabeth Billington at the address given overleaf.

****

The Sixteenth Australasian Conference on Combinatorial Mathematics and Combinatorial Computing will be held at Massey University, 3–7 December, 1990. For further information on this conference, please write to

Dr C.H.C. Little,
Dept of Mathematics and Statistics,
Massey University, Palmerston North, New Zealand.

****

Matters for the Annual General Meeting ...
which will be held on Thursday 13th July at the Fifteenth ACCMC include:

(1) The subscription to the CMSA has remained at $5 for some years, while expenses have risen considerably. It is proposed that this be raised to $10 per year (possibly with a lower amount for students and unemployed or retired people).

(2) At the Annual General Meeting of the CMSA in 1988, the following motion was tabled, to be considered at the AGM in 1989. (See item 6 in the Minutes of the 1988 AGM, in "Combinatorics", Volume 10 (1988), Number 2 (August).)

NOTICE OF INTENTION TO CHANGE THE CONSTITUTION

is hereby given, in advance of the AGM as required. The following motion has been submitted to the Society, for consideration at the AGM:

that, in the item "Termination", the phrase

"to the Australian Mathematical Society"

be changed to

"equally to the Australian Mathematical Society
and the New Zealand Mathematical Society."

Charles Little / Derek Holton

[See page 18 for the Constitution.]

****

RECENT NEWS ITEMS

A D.Sc. was conferred on **Professor Cheryl Praeger** at the University of Western Australia on 18th April, 1989.

Professor R.G. Stanton (University of Manitoba) will receive an honorary D.Sc. from the University of Queensland on 17th August, 1989.

****

THANK YOU ...

We gratefully acknowledge donations for the Fifteenth ACCMCC from **Professor R.G. Stanton** (University of Manitoba) and the Commonwealth Bank, St Lucia, Queensland.

****

DISCRETE MATHEMATICS BOOKLETS

Members are reminded that CMSA aims to publish a series of 30-50 page booklets on discrete mathematics. (See "Combinatorics", Volume 10 (1988), Number 2, August, where even a splendid sample cover page is produced!)

If you wish to contribute please contact the series editor:
Dr. Kevin McAvaney, Department of Computing and Mathematics, Deakin University, Geelong, Vic. 3217, telephone (052) 471276.

****

FIFTEENTH AUSTRALASIAN CONFERENCE
on
COMBINATORIAL MATHEMATICS
and
COMBINATORIAL COMPUTING

The Fifteenth Australasian Conference on Combinatorial Mathematics and Combinatorial Computing will be held at the University of Queensland, Brisbane, Queensland, during the week 10–14 July 1989. All interested persons are cordially invited to attend, and contributed papers are welcome in all areas of combinatorics and combinatorial computing, pure and applied.

Invited speakers include B. Alspach (Simon Fraser), K. Heinrich (Simon Fraser), C.C. Lindner (Auburn), B.D. McKay (A.N.U.), R.C. Mullin (Waterloo), A. Rosa (McMaster), D. R. Stinson (Manitoba), R. Tamassia (Illinois), C. Thomassen (Tech.U.Denmark) and N. Wormald (Auckland).

At least one half–day session will be devoted to combinatorial algorithms. If you have software you are willing to demonstrate at the conference, please get in touch with Dr Peter Eades, Dept of Computer Science, Univ. of Queensland, St Lucia, Qld 4067, and let him know what computing facilities you will need.

The conference registration fee is A$65 (members), A$75 (non–members). There may be a limited amount of money available for students who need financial assistance in order to attend the conference. Any such person should contact Anne Street (address on newsletter cover), and *enclose a recommendation from their supervisor* with their request.

The conference proceedings will be published by **Ars Combinatoria**, subject to the usual arrangements for refereeing. **Deadline** for receipt of manuscripts is August 14th, in camera-ready form; nevertheless all manuscripts will be strictly refereed.

Accommodation is available in Cromwell College, on the University of Queensland campus, at $34 per day for bed and breakfast.

Registration will start on the evening of **Sunday** 9th July, in Cromwell College; wine and cheese will be served. There will be a cocktail party on **Monday** 10th July at the Staff Club after the last lecture of the day, and a film evening at the Schonell Theatre (on campus) on **Tuesday** 11th July, followed by (free!) cake and coffee in the Staff Club. Even if you decide not to attend the film evening (film to be shown will be notified as soon as possible), you are most welcome to join the film-goers for supper in the Staff Club at about 9.15pm, after the film.

The conference excursion, on the afternoon of **Wednesday** 12th July, will be a boat trip on the Brisbane River from the University of Queensland to Newstead House and the new Gateway Bridge, and will include lunch on board and a tour of historical Newstead House. This will cost no more than $20; bring all your friends and it will work out to much less.

The conference dinner will be held on the **Thursday evening, 13th July**, and will take the form of a barbecue. For this we have booked the covered patio at the back of the Staff Club, overlooking the University Lake. If the weather is uncomfortably cold (possible, but not very likely), we'll have access to the Members' Bar at the Staff club. The cost will be $10, for which nuts and chips before dinner, four salads, steak or vegetarian mushroom parcels, coffee and chocolate mints will be provided. Drinks and desserts will be available in addition. The **Annual General Meeting** of the CMSA will be held before the barbecue, after the last lecture on Thursday afternoon.

A survivors' party will be held on the evening of Friday 14th July.

The third registration form is enclosed; please return this if you have not already sent in the second form, and/or you now have extra details to tell us. In particular, if you wish to be met at the airport, please send flight details and we shall try to meet all notified flights; we shall let you know for sure later. Otherwise there is a bus service from the airport to the city of Brisbane, from where either a further bus or a taxi could be taken to the University of Queensland.

A REMINDER! Deadline for abstracts is 10th June 1989.

****

RECENT PUBLICATIONS

Please send in notices of recent publications, preprints and abstracts of theses. Please mark all such material for announcement in the newsletter.

Diane Donovan, *Methods for constructing balanced ternary designs*, Ars Combin. (to appear).

Diane Donovan, *Single laws for subvarieties of squags*, (preprint).

Diane Donovan and Sheila Oates-Williams, *Single laws for sloops and squags*, Discrete Math. (to appear).

Elizabeth J. Billington, *Cyclic balanced ternary designs with block size three and any index*, Ars Combin. (to appear).

Elizabeth J. Billington, *New cyclic* $(61, 244, 40, 10, 6)$ *BIBDs*, Ann. Discrete Math. (to appear).

Elizabeth J. Billington and D.G. Hoffman, *The number of repeated blocks in balanced ternary designs with block size three*, J. Combin. Math. Combin. Computing (to appear).

Elizabeth J. Billington and C.C. Lindner, *Nearly resolvable designs with block size four and $\lambda = 3$*, (submitted).

Elizabeth J. Billington and D.G. Hoffman, *The number of repeated blocks in balanced ternary designs with block size three II*, Discrete Mathematics (to appear).

Elizabeth J. Billington, *Designs with repeated elements in blocks: a survey and some recent results*, (submitted).

Ken Gray, *On the minimum number of blocks defining a design*, Bull. Austral. Math. Soc. (to appear).

D.A. Holton and B.D. McKay, *The smallest non-hamiltonian 3-connected cubic planar graphs have 38 vertices*, J.C.T. B **45** (1988), 305–319.

D.A. Holton and M.D. Plummer, *2-extendability in 3-polytopes*, Colloquia. Math. Soc. Janos Bolyai, 52 Combinatorics (1987), 281–300.

R.E. Aldred, Bau Sheng, D.A. Holton and G. Royle, *An 11-vertex theorem for 3-connected cubic graphs*, J. Graph Theory **12** (1988), 561–570.

Peter Lorimer, *The construction of Tutte's 8-cage and the Condor graph*, (preprint; Department of Math, University of Auckland, Auckland, New Zealand.)

Peter Lorimer, *A construction principle for vertex-transitive graphs*, (preprint; address above.)

Sheila Oates-Williams, *Ramsey varieties of finite groups*, European J. Combin. **9** (1988), 369–373.

Sidney A. Morris, Sheila Oates-Williams and H.B. Thompson, *Locally compact groups with every closed subgroup of finite index*, Bull. London Math. Soc. **19** (to appear).

Jennifer Seberry, *Bhaskar Rao designs of block size 3 over groups of order 8*, Department of Computer Science, University College, The University of N.S.W., Australian Defence Force Academy, Canberra, A.C.T. 2600, Australia; Technical Report CS 88/4.

Gavin Cohen, David Rubie, Jennifer Seberry, Christos Koukouvinos, Stratis Kounias and Mieko Yamada, *A survey of base sequences, disjoint complementary sequences and $OD(4t; t, t, t, t)$*, A.D.F.A. Technical Report CS 88/6.

Michael Newberry and Jennifer Seberry, *User Unique Identification*, A.D.F.A. Technical Report CS88/12.

Thomas Hardjono and Jennifer Seberry, *A Multilevel Encryption Scheme for Database Security*, A.D.F.A. Technical Report CS 88/16.

Christos Koukouvinos, Stratis Kounias and Jennifer Seberry, *Further results on base sequences, disjoint complementary sequences, $OD(4t; t, t, t, t)$ and the excess of Hadamard matrices*, A.D.F.A. Technical Report CS 88/18.

Christos Koukouvinos, Stratis Kounias and Jennifer Seberry, *Supplementary Difference Sets and Optimal Designs*, A.D.F.A Technical Report CS 88/19.

Christos Koukouvinos, Stratis Kounias and Jennifer Seberry, *Further Hadamard Matrices with Maximal Excess and New SBIBD*$(4k^2, 2k^2 + k, k^2 + k)$, A.D.F.A. Technical Report CS 88/20.

Christos Koukouvinos and Jennifer Seberry, *Hadamard Matrices of order* $\equiv 8 (\bmod 16)$ *with Maximal Excess*, A.D.F.A. Technical Report CS 88/21.

R.G. Stanton and Anne Penfold Street, *Further results on minimal defect graphs on seventeen points*, Ars Combinatoria **26A** (1988), 85–90.

Martin J. Sharry and Anne Penfold Street, *Partitioning sets of triples into designs*, Ars Combinatoria **26B** (1988), 51–66.

D.R. Breach and Anne Penfold Street, *Partitioning sets of quadruples into designs II*, J. Combin. Math. Combin. Computing **3** (1988), 41–48.

Martin J. Sharry and Anne Penfold Street, *Partitioning sets of triples into designs II*, J. Combin. Math. Combin. Computing **4** (1988), 53–68.

Anne Penfold Street and Deborah J. Street, *Latin squares and agriculture: the other bicentennial*, The Mathematical Scientist **13** (1988), 48–55.

Martin J. Sharry and Anne Penfold Street, *Partitioning sets of quadruples into designs I*, Discrete Mathematics (to appear).

Martin J. Sharry and Anne Penfold Street, *Partitioning sets of quadruples into designs III*, Discrete Mathematics (to appear).

J.A. Eccleston and Deborah J. Street, *Construction methods for adjusted-orthogonal row-column designs*, Ars Combinatoria (to appear).

W.H. Wilson, Deborah J. Street and D.A. Moelzer, *Hexagonal grids for choice experiments*, J. Combin. Math. Combin. Computing. (to appear).

COMBINATORIAL WORKSHOP

University of Otago

February 1st-18th 1989

The Combinatorial Workshop organised by the Department of Mathematics at the University of Otago was a quite successful event. It attracted many participants from all over the world. Those attending are listed in Appendix 1.

Professor Zhang Ke Min from Nanjing University planned to attend but was only able to arrive in Dunedin after the Workshop was finished.

Several problems were posed. Most of these are contained in Appendix 2. Significant strides were made on Bill Jackson's problem and progress was made on several other problems.

During the workshop the following talks were given:

Brian Alspach	The Double Cover Conjecture
Katherine Heinrich	Shelah's Proof of van der Waerden's Theorem
Charles Little	A Constructive Proof of the 2-Ear Theorem
Peter Lorimer	Symmetry, Groups and Graphs
Ebad Mahmoodian	Designs and Graphs
Brendan McKay	Random Graphs
Gloria Olive	Some Special Functions that arise in Combinatorics
Raymond Scurr	Generalised Powers Revisited

The organisers wish to thank the New Zealand Mathematical Society and the Beverly Fund of the University of Otago for their financial support.

Derek Holton

Appendix 1 (List of Participants)

Appendix 2 (List of Problems)

COMBINATORIAL WORKSHOP

List of Participants

Brian Alspach,	Simon Fraser University, Burnaby, B.C., Canada.
Ernie Cockayne	University of Victoria, P.O. BOX 1700 B.C., Canada
David Glynn	University of Canterbury, Private Bag, Christchurch. (math060@canterbury.ac.nz)
Susan Hamm	Simon Fraser University
Donovan Hare	Simon Fraser University
Katherine Heinrich	Simon Fraser University
Finbarr Holland	University College, Cork, Ireland
Derek Holton	Maths & Stats, University of Otago, P.O. Box 56, Dunedin, N.Z. MATHDAH@OTAGO.AC.NZ
Bill Jackson	University of Auckland, Private Bag Auckland.
Charles Little	Massey University, Private Bag, Palmerston North
Peter Lorimer	University of Auckland.
Dingjun Lou	University of Otago
Brendon McKay	Australian National University, Canberra, ACT, Australia
Ebad Mahmoodian	Sharif University of Technology, P.O. Box 11365-8639, Tehran, Iran.
Susan Marshall	Simon Fraser University
Christine Mynhardt	University of South Africa Box 392, Pretoria, South Africa 0001
Gloria Olive	University of Otago
Akira Saito	Tohoku University, Sendai, Japan (from April 1989, Nihon University, Japan)
Raymond Scurr	University of Otago
Bob Sulanke	Boise State University, Boise, Idaho
Marijke van Rossum	La Salle University, Philadelphia
Ray Watson	University of Melbourne, Parkville, Vic 3052, Australia.
Joseph Yu	Simon Fraser University
Qinglin Yu	Simon Fraser University

COMBINATORIAL WORKSHOP

Problems

Problem 1 (Ernie Cockayne)

The Ramsey Number (classical) $N(q_1, q_2; 2)$ may be defined in terms of independent sets of vertices of graphs as follows: -

$N(q_1, q_2; 2)$ is the smallest n such that in any 2-edge colouring (red-blue) of K_n, the red subgraph has an independent set of size q_1 or the blue subgraph has an independent set of size q_2.

Analogous numbers may be defined using irredundant sets instead of independent sets. We wish to study these new Irredundant Ramsey Numbers and possible generalisations.

Problems 2 & 3 (David Glynn)

Notation:

Symbols ⇔⇒≡∀∃∈→↔Σ

Z the set of integers

Q the set of rational numbers

S set of n elements

m(S) set of subsets of S of size m, (m ∈ Z)

p(S) set of all subsets of S

1. Let G be a permutation group acting on a set S of elements. Define an equivalence relation on p(S) by A ≡ B ⇔ ∃ g ∈ G with $A^g = B$. Then this relation is a geometry in the sense of RoG I. Furthermore, if we define A to be the complementary set of A in S, (∀ A in p(S)), then there is a natural mapping of the associated equivalence classes of these sets a ↔ a, where a and a are the equivalence classes (or subgeometries) of the submodels A and A respectively. Now, a geometry on n elements that satisfies the property that the number of subgeometries of size m is equal to the number of geometries of size n–m, could be called *complementary*. Thus, every permutation group gives a complementary geometry. The question is: what extra conditions are necessary, if any, to ensure that a complementary geometry corresponds to a permutation group?

2. Let H be the geometry corresponding to a permutation group G, acting on a set S, as in 1. above. (Then each subgeometry of H corresponds to an orbit of G acting in the natural way on p(S).) The natural ring (over Z, or algebra over Q), C(H), as defined in RoG I, has basis elements (a), where a is a subgeometry of H, and where the multiplication

(a)(b) := $\Sigma\gamma$(a,b;c)(c), where γ(a,b;c) is the number of ways a and b glue together (with possible overlapping, to give a fixed model of the subgeometry c. Now this ring has a very nice structure given by the direct sum of the coefficient ring (Z or Q), x times, where x is the number of subgeometries of H = the number of orbits of G acting on p(S). The complete set of x mutually orthogonal idempotents of C(H), given in RoG I, which give all information about C(H), is given by the formulae:

$I_c = \Sigma(-1)^{|d|-|c|}$(c,d](d), where (c,d] is the number of submodels of c in a fixed model of d, and where the sum is over all subgeometries of H.

The question involves the interesting fact that yet another ring may be defined in exactly the same way, but with the γ(a,b;c) being replaced by δ(a,b;c): the number of ways a and b glue together to give a fixed model of c, but with the gluing omitting all the elements in the intersection of the models that represent a and b; i.e the symmetric difference, (C = AΔB). The basis elements of this new ring are denoted <d>, for d in H. Thus, <a><b> = <a><b> for all subgeometries a and b of H. One problem is to find the formulae for the complete set of mutually orthogonal idempotents of this new ring ... e.g. $\Sigma<d>/2^n$ is such an idempotent, as is $\Sigma(-1)^{|d|}<d>/2^n$. (Of course, the two rings defined above are isomorphic!) The results obtained may be important for the graph reconstruction conjectures. This second ring or algebra could be called the Δ-ring or Δ-algebra, if no other better names come to mind. This fundamental problem has just been solved by the writer as follows.

For any two subgeometries a,b of H, define <a,b] to be the number of models of a intersecting a fixed model of b in an even number of points, minus the number of models of a intersecting in an odd number. Then the principal idempotent (in the Δ-algebra) corresponding to a, J_a, is given by the formula $J_a = \Sigma<a,b]<b>/2^n$, where the sum is over all subgeometries b in H. The inversion formula is surprisingly similar: $<a> = \Sigma<a,b]\, J_b$. The main question is to use the theory of these rings to prove results about reconstruction and about other problems.

Examples

(a) Let the group G be the symmetric group S_n acting on a set K of size n. Let S = 2(K). Then G acts on p(S) in the natural way, and each orbit of G on p(S) corresponds to the isomorphism class of an edge-graph having n vertices. Thus the geometry H = (S_n,2(K)), defined in(2) above has,as its subgeometries, all the edge-graphs on n vertices. The ring C(H) can be used to prove the best bounds for the edge-graph reconstruction problem (Lovasz and Müller), but a better knowledge of the second ring of (2) may improve this situation further. By the way, what is the status of the reconstruction conjectures for (S_n,m(K)), m > 2, (hypergraphs), or for other permutation groups?

(b) Let n = 3, and let H = (S_3,2(K)), where |K| = 3. Thus the edge-graphs all have 3 vertices. Let a, b, c, d be the edge-graphs with 0, 1, 2, 3 edges respectively. Then we have (in the Δ-algebra): $a^2 = d^2 = a$, $b^2 = c^2 = 3a+2c$, ab = ba = b, ac = ca = c, ad = da = d, bc = cb = 2b+3d, bd = db = c, cd = cd = b; and the principal idempotents are: (a+b+c+d)/8, (a–b+c–d)/8, (3a+b–c–3d)/8, (3a–b–c+3d)/8.

(c) Let n = 4, and let H = (S_4,2(K)), where |K| = 4. Thus the edge-graphs all have 4 vertices. There are 11 such edge-graphs. Check that the empty graph, the graph with 2 disjoint edges, the square, and the complete graph, form a subalgebra of the Δ-algebra which is isomorphic to the Δ-algebra constructed from the edge-graphs on 3 vertices. (See (b) above.) There are other nice subalgebras too. Find them. Look at the pairs of graphs in H that form counter-examples to unique reconstruction.

(d) Exercises (* for perhaps a hard one):

(1) Show that the matrix A of coefficients $<a,b]$ satisfies $A^2 = 2^n I$, where $n = |S|$.

(2) Show that $<a,b] = (-1)^{|a|}<a,b]$, for all a,b in H.

(3) Show that $[a]<a,b] = [b]<b,a]$, for all a,b in H, where $[t]$ is $|Aut(t)|$, for t in H. (Use the fact that the number of models of t in S is $|G|/[t]$.)

(4) Show that $<a,b] = (-1)^{|b|}<a,b]$, for all a,b in H.

(5) Show that the mapping h_b from the Δ-algebra of H to Q defined by $h_b(a) = <a,b]$ is a homomorphism, for all b in H.

(6)* Show that the idempotents J_a are orthogonal and idempotent, (i.e. $J_a . J_b = 0$ or J_a, as $a \neq b$ or $a = b$), and prove the idempotent formulae.

(7)* Suppose that $a \neq b$ are in H, and $|a| = |b| = e$. Suppose also that $(x,a] = (x,b]$ for all x in H with $|x| < e$. The notation $(x,a]$ denotes the number of models of a in a fixed model of b. Thus a and b are *not uniquely reconstructable* in H. Show that $[a]\{I_a - (a)\} = [b]\{I_b - (b)\}$, that $I_a = (a)^2 - [a]^{-1}[b](a)(b)$, and $I_b = (b)^2 - [b]^{-1}[a](a)(b)$, in C(H).
Multiply these formulae by (x) in C(H) and interpret the coefficients to show that $(x,a] - (x,b] = (-1)^e . \{(x,a] - (x,b]\}$. Then show , for any g in H, if the number of models of g in S, $|G|/[g]$, is less that 2^{e-1}, or if $|g| \leq e$ and $g \neq a$ or b, then $(g,a] = (g,b]$. Finally, show Müller's result for edge-graphs (substitute g = a), that if $G = S_n$, (acting on the 2-sets of an n-set), then $[a] \leq n!/2^{e-1}$, i.e. that since $[a] \geq 1$, $n! \geq 2^{e-1}$ is a necessary condition for a and b to exist. Thus a and b have a fairly small number of edges (or elements).
Finally, show that $h_a(x) - h_b(x) = 2^e . \{(x,a] - (x,b]\} = (-2)^e . \{(x,a] - (x,b]\}$, for all x in H.

(e) If G is the identity group, then the Δ-algebra is just the group algebra of $Z_2{}^n$ (the direct product of the cyclic group of order 2, n times), over Q. Then the matrix A of coefficients $<a,b]$ is a Hadamard matrix ("orthogonal" matrix of 1's and –1's).

Further Results

Formulae connecting the permutation group algebras:

Let C(H) be the natural algebra of H (with unions of models), and let D(H) be the Δ-algebra of H (with symmetric differences of models). The subgeometries of H correspond to the orbits on p(S) of a permutation group G acting on S. Let $n = |S|$, let $(a,b]$ be the number of submodels of a in a fixed model of b, and let $<a,b]$ be the number of models of a intersecting a fixed model of b in an even number of elements, minus the number of models intersecting in an odd number, where a and b are subgeometries in H. (The sums are all over subgeometries b in H.)

(1) the idempotents of C(H): $I_a = \Sigma(-1)^{|b|-|a|}(a,b](b)$

(2) $(a) = \Sigma(a,b]I_b$

(3) the idempotents of D(H): $J_a = \Sigma<a,b]<b>/2^n$

(4) $<a> = \Sigma<a,b]\, J_b$

(5) $I_a . I_b = 0$ if $a \neq b$, and $= I_a$ if $a = b$

(6) $J_a . J_b = 0$ if $a \neq b$, and $= J_a$ if $a = b$

(7) ∀ a in H, the mapping g_a: C(H) → Q defined by $g_b((a)) = (a,b]$ is a homomorphism onto Q

(8) ∀ a in H, the mapping h_a: D(H) → Q defined by $h_b(<a>) = <a,b]$ is a homomorphism onto Q

(9) C(H) ≅ D(H); let the isomorphism be $I_a \leftrightarrow J_a$, for all a in H, from now on; i.e. $I_a = J_a$ & $g_a = h_a$

(10) $(a) = \Sigma\, (-1)^{|b|}(a,b]<b>\, /2^{|a|}$

(11) $<a> = \Sigma\, (-2)^{|b|}(a,b](b)$

(12) $(a,y] = \Sigma\, (-1)^{|b|}(a,b]<b,y]\, /2^{|a|}$

(13) $<a,y] = \Sigma\, (-2)^{|b|}(a,b](b,y]$

Reconstruction theory:
Let x and y have the same proper subgeometries;
i.e. $(b,x] = (b,y]$ for all $b \neq x$ or y, and $|x| = |y| = e$.
Then by (13), $\langle a,x] - \langle a,y] = (-2)^e (a,x] - (-2)^e (a,y] = (-2)^e \{(a,x] - (a,y]\}$.
Thus $\langle x,x] - \langle x,y] = (-2)^e \{(x,x] - (x,y]\} = (-2)^e$.
Let X and Y be fixed models of x and y respectively, and let the number of models of x intersecting X in an (even,odd) number and Y in an (even,odd) number be ee, eo, oe, oo respectively.
Then $(ee + eo - oe - oo) - (ee + oe - eo - oo) = (-2)^e$.
Hence $oe - eo = (-2)^{e-1}$, and so $eo + oe \geq 2^{e-1}$.
Thus the number of models of x is at least 2^{e-1} and so $n!/[x] \geq 2^{e-1}$.
This gives Müller's result for H. Notice also that the set of all elements k of C(H) with $g_x(k) = g_y(k)$ is a subring R such that $\Sigma \chi_b \langle b\rangle$ in R $\Leftrightarrow$ $\Sigma \chi_b(b)$ in R.

References:

Avellis, G., & Borzacchini, L., Reconstruction theory for G-graphs, Ars Combinatoria 25A, 173-177, 1988.

Kerber, A., Enumeration under finite group action: symmetry classes of mappings, Lecture Notes in Mathematics 1234, (Combinatoire énumérative, Montreal, 1985), 161-176, 1986.

Krasikov, I. & Roditty, Y., Balance equations for recursive problems, Archiv der Mathematik (Basel) 48 (5), 458-464, 1987.

Müller, V., The edge reconstruction hypothesis is true for graphs with more than $n\log_2 n$ edges, J. Combin. Th. Ser. B 22 (1977), 281-283.

Plesken, W., Counting with groups and rings, J. Reine und Angewandte Math 334, 40-68, 1982.

RoG I: Glynn, D.G., Rings of Geometries I, J. Combin. Th. Series A 44 (1987), 34-48.

RoG II: Glynn, D.G., Rings of Geometries II, J. Combin. Th. Series A 49 (1988), 26-66.

Problem 4 (Derek Holton)

The aim is to eventually find the largest possible k for which the following statement is true:

In any 4-connected 4-regular graph G any k vertices lie on a cycle.

We know that $k \geq 8$. A possible way to show that $k \geq 9$ is to construct a 3-connected 3-regular subgraph of G because it is known that in any such graph any 9 vertices lie on a cycle.

(Actually, if we could construct such a subgraph I think we could probably show $k \geq 12$.)

Problem 5 (Bill Jackson)

Let $f(n,d)$ be the largest integer such that every simple planar graph on n vertices of minimum degree d has a matching M with $|V(M)| \geq f(n,d)$. It is easy to see that $f(n,1) = 2$, and $f(n,2) = 4$ for $n \geq 4$. I think I can show that $f(n,3) = 2(n+4)$. Can one determine $f(n,4)$ and $f(n,5)$?

Problem 6 (Bill Jackson)

Various authors have constructed families of 3-connected planar graphs G whose longest cycle has length n^t for a suitable constant $t < 1$. Can one obtain a reasonable lower bound for the length of a longest cycle in such graphs?

Problem 7 (Charles Little)

Let G be a brick for which every ear decomposition includes more than one 2-ear addition. Must G be isomorphic to the Petersen graph?

(The terminology is taken from Lovasz and Plummer, *Matching Theory* Akademiai Kiado, Budapest, 1986.)

Problem 8 (Chris Malcolm)

A Colourful Solution to the Soma Puzzle

An unkind person recently told me that all 239 solutions (there's a bug in your program I told him - the 240th is weird) to the soma cube puzzle could be generated in a few minutes on a PC in (eeugh!) FORTRAN. Since it takes my PROLOG program several hours to do this, and I will soon discover how much horribly longer it will take to solve soma5 and soma6 problems, I am VERY interested. In order to keep my proposed programme of SERC-funded research fun", it is clear that I need orders of magnitude improvement in the initial solution of the soma part dispositions within the shape. Note that I am cocnerned with arbitrary shapes, not just cubes.

He says it is done by colouring. First you checker the cube (2 colours), and throw out those part positions which don't give the right colouring, i.e., the right totals of cubies of each colour. Especially neat about checkering parts is that the colouring is rotation independent. So far so good. Then you paint the cube with more colours, and more, and more, in the same sort of way (?), until you get to 27 colours, at which point the problem is solved (?!?).

My problem is not being able to understand "in the same sort of way". I see the utility of checkering, which is a well known technique in this kind of problem, but all >2 colour colourings of the cube I have thought of don't seem - to my naive inspection - to have much of a computational edge over "will this part fit in the hole" with backtracking, though they may usefully divide the search space by a few.

My informant has unfortunately forgotten the details - "it was a long time ago", and kept no records "because it was trivial".

Suggestions, clues, pointers to research papers, etc., gratefully received. I now have a few dozen papers on the soma cube puzzle, and its 2D analogue, pentominoes, mostly by mathematicians fascinated by the combinatorial explosion, and none of them have suggested anything beyond checkering (and counting vertices, edges, etc.), so this method (if it exists) is not obvious, and could merit a math paper.

" fun - a technical term in robotics research meaning "industrially relevant".

<u>Problem 9</u> (Robert Sulanke)

Counting lattice paths with respect to the number of certain turns.

Consider the usual lattice paths on Z^2 with step set $\{V,H\} = \{(0,1),(1,0)\}$. Let $P(a,b)$ denote the set of pairs of lattice paths from $(0,0)$ to $(a+1, b+1)$ such that the two paths intersect only at $(0,0)$ and $(a+1, b+1)$. $|P(a,b)|$ are the Narayana or Runyon numbers and $\Sigma_{a+b+1} |P(a,b)|$ are Catalan numbers. Let $P(a,b,s,z)$ be that subset of $P(a,b)$ where the lower path has s VH couples (right-hand turns) and the upper path has z HV couples.

(1) $\left|\bigcup_s P(a,b,s,z)\right| = \frac{1}{z+1}\binom{a}{z}\binom{b}{z}\binom{a+b}{a}$ [Shapiro, 1984, others]

(2) $|P(a,b,s,z)| = \frac{ab(s+z+1)-sz(a+b+1)}{a\ b\ (s+1)\ (z+1)}\binom{a}{s}\binom{b}{s}\binom{a}{z}\binom{b}{z}$ [Sulanke, others?]

Some problems to consider:

1. Interpret $1/(z+1)$ in (1) and the leading quotient in (2).

2. (2) was derived using an invariance property of cyclic permutations of certain other lattice paths, as a bijection exists between the set of these paths and $P(a,b,s,z)$. This method relates to Lagrange inversion. Extend method to other problems.

3. Generalize results to triples of paths in 2 or 3 dimensions.

4. Look for nice q-analogues.

COMBINATORIAL MATHEMATICS SOCIETY OF AUSTRALASIA

CONSTITUTION

Aim:	To promote Combinatorial Mathematics in Australasia.
Membership:	Ordinary Membership is open to anyone paying the prescribed membership fee.
Fees:	These are to be determined at the Annual Meeting of the Society.
Annual Meeting:	This will be held at the Annual Conference of the Society. An audited account of the financial transactions of the Society for the previous calendar year, must be given at the Annual meeting.
Officers:	The Annual Meeting shall elect a Director, Secretary, Treasurer and other members of the Committee of the Society as it sees fit.
Committee:	The Committee of the Society shall be responsible to the Society for: (1) the general business of the Society; and (2) the organisation of the next Annual Conference. The Committee shall have power to coopt.
Voting:	A quorum at a meeting of the Society shall consist of ten members. Any contentious issue shall be determined by a simple majority vote of the members present, except the alteration of the Constitution or the termination of the Society, both of which require a two thirds majority. Notice of intent to change the Constitution or to terminate the Society must be posted to the Membership one calendar month before the Annual Meeting or a meeting specially convened for one of those purposes.
Termination:	In the event of the Society being terminated, all residual funds and assets of the Society shall be donated to the Australian Mathematical Society.

Fifteenth Australasian Conference
on
Combinatorial Mathematics
and
Combinatorial Computing

10th–14th July, 1989
University of Queensland

Third and final Circular
Registration Form

If you are coming to the conference and have not returned the second circular, or think you have **new** information for us, please fill in this form and return it to:

The Secretary, CMSA
Department of Mathematics
University of Queensland
St. Lucia
Queensland, Australia 4067.

Name (title) and address:

Do you wish to give a talk?

If so, title:

*** **Deadline** for abstracts (max length 150 words) is **10th June.**

Do you want accommodation in Cromwell College?
If so, for how many people and for which nights?

** **Arrival date and time** (and flight details if you hope to be met)
**
** **Departure date (and time):**

Do you wish to go on the river excursion?

Do you wish to attend the conference BBQ dinner?

Are you interested in the film evening? (There will be a free light supper afterwards at 9pm at the Staff Club.)